Two star polar alignment

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October 7, 2003

This paper documents a method to polar align a telescope using observations of two stars. For telescopes such as the Meade LX200, this can provide a fast method of alignment and an assessment of how accurate an alignment is.

We assume the standard symbols:

$$\begin{gathered}\alpha = \textit{Right Ascension}\\ \delta = \textit{Declination}\\ \phi = \textit{Site Latitude}\\ h = \textit{Hour Angle } (= \alpha - \textit{Local Sidereal Time})\end{gathered} \tag{1}$$

We introduce some new symbols:

$$\begin{gathered}\Delta\alpha = \textit{Error in Right Ascension}\\ \Delta\delta = \textit{Error in Declination}\\ \Delta e = \textit{Alignment error in elevation (altitude)}\\ \Delta a = \textit{Alignment error in azimuth}\end{gathered} \tag{2}$$

In this case a positive error in elevation means that the polar axis of the telescope is pointed above the northern celestial pole. A positive error in azimuth means that the polar axis of the telescope is pointed to the east of the northern celestial pole.

The fundamental equations are:

$$\begin{gathered}\Delta\alpha = \Delta e \tan\delta \sin h + \Delta a(\sin\phi - \cos\phi \tan\delta \cos h)\\ \Delta\delta = \Delta e \cos h + \Delta a \cos\phi \sin h\end{gathered} \tag{3}$$

These equations relate errors in elevation and azimuth to errors in Right Ascension and Declination. Please note that errors in Right Ascension are equal to errors in Hour Angle. Please make sure you do the proper angle conversions for the units you are using. Finally, p.111 of “Telescope Control’ by Trueblood and Genet has equations similar to those of (3). However, they are missing the sin(latitude) term in the equation for the error in RA. I believe (3) is correct because of the following simple example. Suppose a telescope is exactly at a pole and suppose there is no elevation error. Suppose further that the telescope azimuth is misaligned by some angle. Four of the five terms in (3) are zero either due to the elevation error being 0 or the latitude being 90 or -90 so the cosine term for latitude is 0. Clearly this configuration means that there is an error in RA or HA and it is equal to the misalignment amount. This is exactly the term in (3) that is missing from Trueblood and Genet.

Part of the complication with polar alignment are the biases in angle readouts that are present at the start of alignment. This is part of the problem to be resolved during the alignment procedure.

The suggested procedure is to:

1. Pick two stars to use for alignment. As we see below, there are some constraints on the choice of stars. A bad choice will not allow solving for one of the two misalignment parameters,) e or) a.

2. Go to the first star and adjust the Right Ascension and the Declination readouts to match that of the selected star. (This is easy with a Meade LX200: you sync on the star.)

3. Go to the second star and observe the error in Right Ascension and the error in Declination. The difference is computed as the observed location minus the true location. Note that this difference removes the effect of the initial biases in the angles. Using subscript 1 for the first star and subscript 2 for the second star, the observed errors and the misalignment errors are related by:

$$\begin{bmatrix}\Delta\alpha \\ \Delta\delta\end{bmatrix} = \begin{bmatrix}\tan\delta_2 \ \sin h_2 - \tan\delta_1 \ \sin h_1 & \cos\phi(\tan\delta_1 \ \cos h_1 - \tan\delta_2 \ \cos h_2) \\ \cos h_2 - \cos h_1 & \cos\phi(\sin h_2 - \sin h_1)\end{bmatrix} \begin{bmatrix}\Delta e \\ \Delta a\end{bmatrix} \qquad \textbf{(4)}$$

Now the determinant of the matrix is:

$$determinant = d = \cos\phi(\tan\delta_1 + \tan\delta_2)(1 - \cos(h_1 - h_2)) \qquad \textbf{(5)}$$

So the inverse relationship is:

$$\begin{bmatrix}\Delta e \\ \Delta a\end{bmatrix} = \begin{bmatrix}\cos\phi(\sin h_2 - \sin h_1)/d & -\cos\phi(\tan\delta_1 \ \cos h_1 - \tan\delta_2 \ \cos h_2)/d \\ (\cos h_1 - \cos h_2)/d & (\tan\delta_2 \ \sin h_2 - \tan\delta_1 \ \sin h_1)/d\end{bmatrix} \begin{bmatrix}\Delta\alpha \\ \Delta\delta\end{bmatrix} \qquad \textbf{(6)}$$

4. Using the errors from step 3 and equations (5) and (6), compute Δe and Δa.

5. Now use the mechanical adjustments to move the scope (using star motion in the field of view, etc).

6. I prefer to now pick a third star near the meridian and near a declination of $\frac{\phi - 90}{2}$.

Using the star's Right Ascension, compute its Hour Angle. Using the Hour Angle and Declination as h_2 and δ_2 in Equation (4), compute the expected error in Right Ascension and Declination and subtract this from the star's true Right Ascension and Declination. Move the scope to the computed coordinates.

7. I then use mechanical adjustments (that is, not the polar axis or the declination axis) to center the star.

Assume for example

Site latitude is 42° 40' N

star 1 has an hour angle of 3^h and a declination of 48°

star 2 has an hour angle of 23^h and a declination of 45°

RA error of -12 arc minutes at star 2 (error at star 1 is 0 because of step 2)

Dec error of -21 arc minutes at star 2 (error at star 1 is 0 because of step 2)

Then we can compute:

the computed error in elevation (altitude) is 7.39 arc minutes and the error in azimuth is 32.26 arc minutes.

It is pretty easy to do the alignment (except for the calculations). I have done two iterations of this procedure and have gotten computed alignment errors of less than one arc minute. I will typically choose the first star at an hour angle of 3 hours (or - 3 hours) and declination of about 43°. I then choose the second star near the zenith. Since my latitude is about 43°, this means that the determinant, Equation (5), is about 0.4. Note that you should avoid stars with similar hour angles, stars close to the celestial equator, and stars whose declinations are close to negatives of each other. Each of these factors makes the determinant 0 or close to 0. This means that Equation (6) becomes unstable.

I have written a program that will use a RS-232 link to a Meade LX200, reads the locations of the first and the second star, match them with stars in the 250 star Meade database, and then computes the misalignment errors. (It also selects the third star, and slews to its location).

The 'magic' number for the declination of the star is the point where true angular motion in azimuth matches true angular motion in Right Ascension.

I have thought in detail about what this means in the southern hemisphere. I do know that the 'magic' declination is $\frac{90+\phi}{2}$. I also know that a positive azimuth misalignment indicates that the telescope polar axis is WEST of the South Celestial Pole. What I do not know, is what a positive elevation misalignment means (I believe that it means the polar axis is below the South Celestial Pole).

There are some 'extra' errors that can be introduced during the process and should be considered. The two major errors are refraction and mirror shift. With care these can be minimized. For example, pick stars above 45 degrees and on the same side of the meridian (for the first two stars). Of course, accuracy of angle readouts will affect the resulting accuracy of the estimates of misalignment.

The advantage of this method over the drift method of alignment is speed. With experience a user can get three iterations in in less than 10 minutes (assuming a programmable calculator or computer to help with the calculations). The disadvantage of this method over the drift method of alignment is that some calculation is required and accuracy of readouts of Right Ascension and of Declination could affect the accuracy estimates of the alignment shift.

Please feel free to forward any comments to me at rppass@rppass.com.

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